Phase Transitions in Condensed Matter and Relativistic QFT *

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Kibble and Zurek have provided a unifying picture for the onset of phase transitions in relativistic QFT and condensed matter systems respectively, strongly supported by agreement with condensed matter experiments in ^3He. The failure of a recent experiment on ^4He to agree with the predictions has prompted a reappraisal of this picture. We provide an alternative picture that explains the experimental evidence.

I. THE ONSET OF PHASE TRANSITIONS

The two environments in which thermal field theory is relevant are the early universe and heavy-ion collisions, in both of which there are changes of phase. Not surprisingly, one of the themes of this series of Workshops on Thermal Field Theory (TFT) has been phase transitions.

Consider a situation in which the symmetry group of the theory is broken, on cooling through the critical temperature T_c, by its degenerate groundstate manifold. Using the tools of *equilibrium* TFT we can determine the nature of the transition. At late times after the transition the fields are ordered on large scales, in that the field will adopt a single value from this degenerate set over a large spatial region. This might seem to be all that we need to know, but how this is achieved can have interesting consequences.

Such problems require that we go beyond equilibrium TFT. In practice, we often know remarkably little about the dynamics of thermal systems. For simplicity, I shall assume scalar field order parameters, with *continuous* transitions. In particular, I want to discuss some aspects of the *onset* of such phase transitions, the very early times after the implementation of a transition when the scalar fields are only just beginning to become ordered.

A simple question to ask is the following: In principle, the field correlation length diverges at a continuous transition. In practice, it does not. What happens? This is relevant for transitions that leave topological defects like walls, monopoles, vortices, or textures in their wake since we might expect 'defects' to be just that, entities whose separation is characterised by the correlation length. If this were simply so, a measurement of defect densities would be a measurement of correlation lengths. Estimates of this early field ordering have been made by Kibble [1,2], using simple causal arguments, because of the implication of defects for astrophysics. Vortices, in particular (cosmic strings), can be important for structure formation in the universe. [1]

There are great difficulties in converting predictions for the early universe into experimental observations. Zurek suggested [3] that similar arguments were applicable to condensed matter systems for which direct experiments on defect densities could be performed. This has lead to a burst of activity from theorists working on the boundary between QFT and Condensed Matter theory and from condensed matter experimentalists. To date almost all experiments have involved superfluids, with scalar order parameters of the type considered above and all but one experiment is in agreement with these simple causal predictions.

In this talk I shall

- review the Kibble/Zurek causality predictions for initial correlation lengths and defect densities and summarise the results of the condensed matter experiments.
- present an alternative picture for the onset of defect production, applicable to both relativistic quantum fields and condensed matter systems.
- show how this alternative picture gives essentially the same results as the Zurek picture for those condensed matter systems for which there is experimental agreement.
- provide an explanation for why some condensed matter experiments will be in disagreement with Zurek's predictions, including the experiment in question.
- try to draw some broader conclusions about the relevance of Kibble's predictions for phase transitions in QFT.

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[1] In a different context, the topological textures of the NLSM $O(4)$ theory can be identified with Skyrmionic hadrons produced in the hadronisation of quark-gluon plasma.

II. WHEN SYMMETRY BREAKS, HOW BIG ARE THE SMALLEST IDENTIFIABLE PIECES?

Insofar that defect density can be correlated simply to correlation length, the *maximum* density (an experimental observable in condensed matter systems, although not for the early universe) will be associated with the *smallest* identifiable correlation length in the broken phase once the transition has been effected. This provides the initial condition for the evolution of field ordering. In order to see how to identify these 'smallest pieces'[2] it is sufficient to consider the simplest theory, that of a single relativistic quantum scalar field in three spatial dimensions with a double well potential, undergoing a temperature quench. In the first instance we assume that the qualitative dynamics of the transition are conditioned by the field's *equilibrium* free energy, of the form

$$F(T) = \int d^3x \left(\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2(T)\phi^2 + \frac{1}{4}\lambda\phi^4\right). \quad (1)$$

Prior to the transition, at temperature $T > T_c$, the critical temperature, $m(T) > 0$ plays the role of an effective 'plasma' mass due to the interactions of ϕ with the heat-bath, which includes its own particles. After the transition, when T is effectively zero, $m^2(0) = -M^2 < 0$ enforces the Z_2 symmetry-breaking, with field expectation values $\langle\phi\rangle = \pm\eta$, $\eta^2 = M^2/\lambda$. The change in temperature that leads to the change in the sign of m^2 is most simply understood as a consequence of the system expanding. Thus, in the early universe, a weakly interacting relativistic plasma at temperature $T \gg M$ has an entropy density $s \propto T^3$. As long as thermal equilibrium can be maintained, constant entropy S per comoving volume, $S \propto sa(t)^3$, gives $T \propto a(t)^{-1}$ and falling, for increasing scale factor $a(t)$. We shall be too simple in our discussion to warrant the inclusion of a metric in Eq.1.

Some caution is needed in interpreting $V(\phi) = m^2(T)\phi^2/2 + \lambda\phi^4/4$ as the conventional Effective Potential since this is a zero-momentum, or infinite-volume field average, construct. At the very least, a more realistic potential for dynamics would be coarse-grained to the relevant scale. In practice, as long as we never look on scales smaller that the field correlation length this turns out not to matter much. The only relevant point is that $m^2(T)$ vanishes at $T = T_c$, true however $V(\phi)$ is defined.

The purpose of $V(\phi, T)$ is, initially, to set the scale of the fluctuations of ϕ about $\phi = 0$, the subsequent false vacuum. The *equilibrium* correlation length of the field fluctuations is $\xi_{eq}(T) = |m^{-1}(T)|$. It is sufficient to adopt a mean-field approximation in which $m^2(T) \propto (T - T_c)$. For fields averaged on the scale of $\xi_{eq}(T)$ we find $\langle\phi^2\rangle = O(m(T)T)$, much less than η^2 for our relativistic theory in which $T_c = O(\eta)$. After the transition the field, at each point in space, begins to collapse to one of the false vacua $\phi = \pm\eta$, $\eta^2 = M^2/\lambda$, as signalled by $V(\phi)$, initially in a random way. However, the field soon begins to become organised into 'domains', in each of which the field has a constant sign. The defects of this simple model are domain walls, across which the field flips from one minimum to the other. Defect density then looks to be related to domain size.

How this collapse takes place determines the size of the first identifiable domains. It was suggested by Kibble and Zurek that this size is essentially the field correlation length ξ_{eq} at some appropriate temperature close to the transition. I shall argue later that this is too simple but, nonetheless it is a plausible starting point. Near the continuous transition ξ_{eq} becomes large, approaching infinity at T_c. In practice physical correlations remain finite. The question then becomes: how large does the field correlation get in practice? Two very different mechanisms have been proposed for estimating this size.

A. Thermal activation

In the early work on the cosmic string scenario it was assumed [1] that initial domain size was fixed in the Ginzburg regime. By this we mean the following. Suppose the temperature $T(t)$ varies sufficiently slowly with time t that it makes sense to replace $V(\phi, T)$ by $V(\phi, T(t))$. Well away from the transition this is justified, but close to the transition it is not. A band in temperature about T_c in which it is not sensible is $T_G^- < T < T_G^+$, where $T_G^{\pm}$ are the Ginzburg temperatures. As we cool down in the vicinity of T_c, $V(\phi, T)$ ceases to be a reliable guide to the field fluctuations at T_G^+, where the relative Ginzburg temperature is of order of magnitude $|1 - T_G^+/T_c| = O(\lambda)$. Once we are below T_c, and the central hump in $V(\phi, T(t))$ is forming, T_G^- signals the temperature above which there is a significant probability for thermal fluctuations over the central hump on the scale of the correlation length. Most simply, it is determined by the condition

$$\Delta V(T_G^-)\xi_{eq}^3(T_G^-) \approx T_G^- \quad (2)$$

where $\Delta V(T)$ is the difference between the central maximum and the minima of $V(\phi, T)$. Again we find $|1 - T_G^-/T_c| = O(\lambda)$.

Whereas, above (T_G^-) there will be a population of 'domains', fluctuating in and out of existence, at temperatures below T_G^- fluctuations from one minimum to the other become increasingly unlikely. When this happens the correlation length is

$$\xi_{eq}(T_G^-) = O\left(\frac{\xi_0}{\sqrt{\lambda}}\right), \quad (3)$$

where $\xi_0 = M^{-1}$ is the natural unit of length, the Compton wavelength of the ϕ particles.

[2] The title of this section is essentially that posed in recent papers by Zurek [4].

It is tempting to identify $\xi_{eq}(T_G^-)$ with the scale at which stable domains begin to form. Subsequent work shows this to be incorrect. Thermal fluctuations *are* relevant to the formation of small domains, and to wiggles in the boundaries of larger domains, but not in the formation of larger domains themselves. That is an issue that requires more than equilibrium physics. The most simple dynamical arguments can be understood in terms of causality.

B. Causality

If the Ginzburg criteria attempt to set scales once the critical temperature has been *passed*, causal arguments attempt to set scales *before* it is reached. We have seen that $\xi_{eq}(T(t))$ diverges at $T(t) = T_c$, which we suppose happens at $t = 0$. This cannot be the case for the true correlation length $\xi(t)$, which can only grow so far in a finite time. Initially, for $t < 0$, when we are far from the transition, we again assume effective equilibrium, and the field correlation length $\xi(t)$ tracks $\xi_{eq}(T(t))$ approximately. However, as we get closer to the transition $\xi_{eq}(T(t))$ begins to increase arbitrarily fast. As a crude upper bound, the true correlation length fails to keep up with $\xi_{eq}(T(t))$ by the time $-t_C$ at which ξ_{eq} is growing at the speed of light, $d\xi_{eq}(T(-t_C))/dt = 1$. It was suggested by Kibble [2] that, once we have reached this time $\xi(t)$ *freezes* in, remaining approximately constant until the time $t \approx +t_C$ after the transition when is once again becomes comparable to the now *decreasing* value of ξ_{eq}. The correlation length $\xi_{eq}(t_C) = \xi_{eq}(-t_C)$ is argued to provide the scale for the minimum domain size *after* the transition.

Specifically, if we assume a time-dependence $m^2(t) = -M^2 t/t_Q$ in the vicinity of $t = 0$, when the transition begins to be effected, then the causality condition gives $t_C = t_Q^{1/3}(2M)^{-2/3}$. As a result,

$$M\xi_{eq}(t_C) = (M\tau_0)^{1/3}, \tag{4}$$

which, with condensed matter in mind, we write as

$$\xi_{eq}(t_C) = \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^{1/3} \tag{5}$$

where $\tau_0 = \xi_0 = M^{-1}$ are the natural time and distance scales. In contrast to Eq.3, Eq.5 depends explicitly on the quench rate, as we would expect. For $\tau_Q \gg \tau_0$ the field is correlated on a scale of many Compton wavelengths.

C. QFT or Condensed Matter

This approach of Kibble was one of the motivations for a similar analysis by Zurek [3] of transitions with scalar order parameters in condensed matter. Qualitatively, neither the Ginzburg thermal fluctuations, with fluctuation length Eq.3, nor the simple causal argument above depend critically on the fact that the free energy Eq.1 is originally assumed to be derived from a relativistic *quantum* field theory. After rescaling, F could equally well be the Ginzburg-Landau free energy

$$F(T) = \int d^3x \left(\frac{\hbar^2}{2m}(\nabla\phi)^2 + \alpha(T)\phi^2 + \frac{1}{4}\beta\phi^4\right) \tag{6}$$

for a non-relativistic condensed matter field, in which $\alpha(T) \propto m^2(T)$ vanishes at the critical temperature T_c. The only difference is that, in the causal argument, the speed of light should be replaced by the speed of sound, with different critical index.

Explicitly, let us assume the mean-field result $\alpha(T) = \alpha_0\epsilon(T_c)$, where $\epsilon = (T/T_c - 1)$, remains valid as T/T_c varies with time t. In particular, we first take $\alpha(t) = \alpha(T(t)) = -\alpha_0 t/\tau_Q$ in the vicinity of T_c. Then the fundamental length and time scales ξ_0 and t_0 are given from Eq.6 as $\xi_0^2 = \hbar^2/2m\alpha_0$ and $\tau_0 = \hbar/\alpha_0$. It follows that the equilibrium correlation length $\xi_{eq}(t)$ and the relaxation time $\tau(t)$ diverge when t vanishes as

$$\xi_{eq}(t) = \xi_0\left|\frac{t}{\tau_Q}\right|^{-1/2},\ \tau(t) = \tau_0\left|\frac{t}{\tau_Q}\right|^{-1}. \tag{7}$$

The speed of sound is $c(t) = \xi_{eq}(t)/\tau(t)$, slowing down as we approach the transition as $|t|^{1/2}$. The causal counterpart to $d\xi_{eq}(t)/dt = 1$ for the relativistic field is $d\xi_{eq}(t)/dt = c(t)$. This is satisfied at $t = -t_C$, where $t_C = \sqrt{\tau_Q\tau_0}$, with corresponding correlation length

$$\xi_{eq}(t_C) = \xi_{eq}(-t_C) = \xi_0\left(\frac{\tau_Q}{\tau_0}\right)^{1/4}. \tag{8}$$

(cf. Eq.5). A variant of this argument that gives essentially the same results is obtained by comparing the quench rate directly to the relaxation rate of the field fluctuations. We stress that, yet again, the assumption is that the length scale that determines the initial correlation length of the field freezes in *before* the transition begins. Whatever, the field is already correlated on a scale of many Compton wavelengths when it begins to unfreeze.

III. EXPERIMENTS

The end result of the simple causality arguments is that, both for QFT and condensed matter, when the field begins to order itself its correlation length has the form

$$\xi_{eq}(t_C) = \xi_0\left(\frac{\tau_Q}{\tau_0}\right)^{\gamma}. \tag{9}$$

for appropriate γ. [3]

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[3] In fact, the powers of Eq.5 and Eq.8 are mean-field results, changed on implementing the renormalisation group.

For a relativistic *complex* scalar field ϕ with a global $U(1)$ or $O(2)$ symmetry the causality argument is the same, but the physical situation is very different. Consider the theory controlled by a free energy

$$F(T) = \int d^3x \left(|\nabla\phi|^2 + m^2(T)|\phi|^2 + \lambda|\phi|^4 \right) \qquad (10)$$

with $m^2(T)$ switching from positive to negative. The minima of the final potential of Eq.10 now constitute the circle $\phi = \eta e^{i\alpha}$. When the transition begins ϕ begins to fall into the valley of the potential, choosing a random phase. This randomly chosen phase will vary from point to point. Solutions to $\delta F/\delta\phi = 0$ include vortices, topological defects, tubes of 'false' vacuum $\phi \approx 0$, around which the field phase changes by $\pm 2\pi$. In an early universe context these are 'cosmic strings', but we shall not consider their properties here. The jump that Kibble made was to assume that the correlation length Eq.5, equally applicable to a complex field, also sets the scale for the typical minimum intervortex distance.

That is, the *initial* vortex density n_{def} is

$$n_{def} = O\left(\frac{1}{(\xi_{eq}(t_C))^2}\right) = O\left(\frac{1}{\xi_0^2}\left(\frac{\tau_0}{\tau_Q}\right)^{2\gamma}\right). \qquad (11)$$

for $\gamma = 1/3$. Equivalently, the length of vortices in a box volume v is $O(n_{def}v)$. We stress that this assumption is *independent* of the argument that lead to Eq.5.

Since ξ_0 also measures cold vortex thickness, $\tau_Q \gg \tau_0$ corresponds to a measurably large number of widely separated vortices. If it could be argued that this initial network behaves classically then, thereafter, the density will reduce due to the collapse of small loops, intersections chopping off loops which in turn collapse, and vortex straightening so as to reduce the gradient energy of the field.

Even if cosmic strings were produced in so simple a way in the very early universe it is not possible to compare the density Eq.11 with experiment. It was Zurek who first suggested that this causal argument for defect density be tested in condensed matter systems, particularly in liquid helium.

A. Superfluid helium

Vortex lines in both superfluid ^4He and ^3He are good analogues of global cosmic strings. A crude but effective model is to treat the system as composed of two fluids, the normal fluid and the superfluid, which has zero viscosity. In ^4He the bose superfluid is characterised by a complex field ϕ, whose squared modulus $|\phi|^2$ is the superfluid density. The superfluid fraction is unity at absolute zero, falling to zero as the temperature rises to the lambda point at 2.17K. The Landau-Ginzburg theory for ^4He has, as its free energy,

$$F(T) = \int d^3x \left(\frac{\hbar^2}{2m}|\nabla\phi|^2 + \alpha(T)|\phi|^2 + \frac{1}{4}\beta|\phi|^4 \right), \qquad (12)$$

the scaled counterpart of Eq.10. The static classical field equation $\delta F/\delta\phi = 0$ has vortex solutions as before, with width ξ_0.

The situation is more complicated, but more interesting, for ^3He, which becomes superfluid at the much lower temperature of 2 mK. The reason is that the ^3He is a *fermion.* Thus the mechanism for superfluidity is very different from that of ^4He. Somewhat as in a BCS superconductor, these fermions form the counterpart to Cooper pairs. However, whereas the (electron) Cooper pairs in a superconductor form a ^1S state, the ^3He pairs form a ^3P state. The order parameter $A_{\alpha i}$ is a complex 3×3 matrix $A_{\alpha i}$. There are two distinct superfluid phases, depending on how the $SO(3)\times SO(3)\times U(1)$ symmetry is broken. If the normal fluid is cooled at low pressures, it makes a transition to the ^3He-B phase, in which $A_{\alpha i}$ takes the form $A_{\alpha i} = R_{\alpha i}(\omega)e^{i\Phi}$, where R is a real rotation matrix, corresponding to a rotation through an arbitrary ω [5]

The Landau-Ginzburg free energy is, necessarily, more complicated, but the effective potential $V(A_{\alpha i}, T)$ has the diagonal form $V(A,T) = \alpha(T)|A_{ai}|^2 + O(A^4)$ for small fluctuations, and this is all that we need for the production of vortices at very early times. Thus the Zurek analysis leads to the prediction Eq.11, as before, for appropriate γ. However, for ^3He the mean-field approximation is good and the mean-field critical index $\gamma = 1/4$ is not renormalised, whereas for ^4He a better value is $\gamma = 1/3$.

B. Experiments in ^3He.

Although ^3He is more complicated to work with, the experiments to check Eq.11 are cleaner for both experimental and theoretical reasons. First, because the nucleus has spin 1/2, even individual vortices can be detected by magnetic resonance. Second, because vortex width is many atomic spacings the Landau-Ginzburg theory is good.

So far, experiments have been of two types. In the Helsinki experiment [6] superfluid ^3He in a rotating cryostat is bombarded by slow neutrons. Each neutron entering the chamber releases 760 keV, via the reaction $n +^3 He \to p +^3 He + 760keV$. The energy goes into the kinetic energy of the proton and triton, and is dissipated by ionisation, heating a region of the sample above its transition temperature. The heated region then cools back through the transition temperature, creating vortices. Vortices above a critical size (dependent on the angular velocity of the cryostat) grow and migrate to the centre of the apparatus, where they are counted by an NMR absorption measurement. Suffice to say that the quench is very fast, with $\tau_Q/\tau_0 = O(10^3)$. Agreement

with Eq.11 and Eq.8 is very good, at the level of an order of magnitude.

The second type of experiment has been performed at Grenoble and Lancaster [7]. Rather than count individual vortices, the experiment detects the total energy going into vortex formation. As before, ^3He is irradiated by neutrons. After each absorption the energy released in the form of quasiparticles is measured, and found to be less than the total 760 keV. This missing energy is assumed to have been expended on vortex production. Again, agreement with Zurek's prediction Eq.11 and Eq.8 is good.

C. Experiments in ^4He.

The experiments in ^4He, conducted at Lancaster, follow Zurek's original suggestion. The idea is to expand a sample of normal fluid helium, in a container with bellows, so that it becomes superfluid at essentially constant temperature. That is, we change $1-T/T_c$ from negative to positive by reducing the pressure, thereby increasing T_c. As the system goes into the superfluid phase a tangle of vortices is formed, because of the random distribution of field phases. The vortices are detected by measuring the attenuation of second sound within the bellows. Second sound scatters off vortices, and its attenuation gives a good measure of vortex density. A mechanical quench is slow, with τ_Q some tens of milliseconds, and $\tau_Q/\tau_0 = O(10^{10})$. Two experiments have been performed [8,9]. In the first fair agreement was found with the prediction Eq.11, although it was not possible to vary τ_Q. However, there were potential problems with hydrodynamic effects at the bellows, and at the capillary with which the bellows were filled. A second experiment, designed to minimise these and other problems has failed to see any vortices whatsoever.

There is certainly no agreement, in this or any other experiment on ^3He, with the thermal fluctuation density that would be based on Eq.3.

IV. THE KIBBLE-ZUREK PICTURE FOR THE FREEZING IN OF ξ IS CORRECT.

We are in the strange position that, if the second Lancaster experiment [9] is correct, either both the Zurek predictions Eq.11 and Eq.8 works well, or one or both fail. We shall now argue that the freezing in of ξ as in Eq.8 is correct, essentially on dimensional grounds. To show this we need a concrete model for the dynamics.

A. Condensed matter: the TDLG equation

We assume that the dynamics of the transition can be derived from the explicitly time-dependent Landau-Ginzburg free energy

$$F(t) = \int d^3x \, \left(\frac{\hbar^2}{2m}|\nabla\phi|^2 + \alpha(t)|\phi|^2 + \frac{1}{4}\beta|\phi|^4 \right). \quad (13)$$

In (13) $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ is the complex order-parameter field, whose magnitude determines the superfluid density. In equilibrium at temperature T, in a mean field approximation, the chemical potential $\alpha(T)$ takes the form $\alpha(T) = \alpha_0\epsilon(T_c)$, where $\epsilon = (T/T_c - 1)$. In a quench in which T_c or T changes it is convenient to shift the origin in time, to write ϵ as

$$\epsilon(t) = \epsilon_0 - \frac{t}{\tau_Q}\theta(t) \quad (14)$$

for $-\infty < t < \tau_Q(1+\epsilon_0)$, after which $\epsilon(t) = -1$. ϵ_0 measures the original relative temperature and τ_Q defines the quench rate. The quench begins at time $t = 0$ and the transition from the normal to the superfluid phase begins at time $t = \epsilon_0\tau_Q$.

Motivated by Zurek's later numerical [4] simulations with the time-dependent Landau-Ginzburg (TDLG) equation for F of (13), we assume a linear response

$$\frac{1}{\Gamma}\frac{\partial\phi_a}{\partial t} = -\frac{\delta F}{\delta\phi_a} + \eta_a, \quad (15)$$

where η_a is Gaussian noise. More details are given elsewhere [10]. We can show self-consistently that, for the relevant time-interval $-t_C \leq \Delta t \leq t_C$ the self-interaction term can be neglected ($\beta = 0$). This both preserves Gaussian field fluctuations and leads to $\xi \approx \xi_0(\tau_Q/\tau_0)^{1/4}$ arising in a natural way, as we shall see.

At relative time Δt the diagonal *equal-time* correlation function $G(r,\Delta t)$ is defined by

$$\langle\phi_a(\mathbf{r})\phi_b(\mathbf{0})\rangle_{\Delta t} = \delta_{ab}G(r,\Delta t). \quad (16)$$

Anticipating that correlation lengths are approximately frozen in during this period it is sufficient to perform our calculations at $\Delta t = 0$. Eq.15 is solvable, in time and space units τ_0 and ξ_0, to give the Fourier transform of $G(r,\Delta t)$ as

$$G(k) = e^{\tau_Q k^4}\int_{\tau_Q k^2}^{\infty} dt \, e^{-t^2/\tau_Q}, \quad (17)$$

independent of ϵ_0, provided $\epsilon_0\tau_Q$, $\epsilon_0^2\tau_Q \gg 1$. For ^4He, where this is more problematical, this is valid, since although $\epsilon_0 \sim 10^{-2} - 10^{-3}$ is very small, $\tau_Q \sim 10^{10}$ is so large.

On dimensional grounds the correlation length of the field is $O(\xi_{eq}(t_C)) = O(\xi_0(\tau_Q/\tau_0)^{1/4})$. However, rather than have an *asymptotic* fall-off of the form $e^{-r/\xi_{eq}(t_C)}$ for large r, $G(r) \propto \exp(-O((r/\xi_{eq}(t_C))^{4/3}))$. Nonetheless, numerically, it is remarkably well represented by $e^{-r/\xi_{eq}(t_C)}$, with coefficient *unity* in the exponent, for r being a few multiples of $\xi_{eq}(t_C)$, for reasons that are not clear to us. In that sense Zurek's prediction is robust, since explicit calculation [21] shows that $\xi(t)$ does not vary strongly in the interval $-t_C \leq \Delta t \leq t_C$.

B. QFT: the free roll

The dynamical equations for a hot quantum field (Heisenberg equations with Boltzmann boundary conditions) are very different from the empirical TDLG equation above. Fortunately, as for the condensed matter case, the interval $-t_C \leq \Delta t \leq t_C$ occurs in the *linear* regime, when the *operator* equations

$$\frac{\partial^2 \phi_a}{\partial t^2} = -\frac{\delta F}{\delta \phi_a}, \tag{18}$$

for F of Eq.10, are solvable in terms of the mode functions $\chi_k^{\pm}(t)$, satisfying

$$\left[\frac{d^2}{dt^2} + \mathbf{k}^2 + m^2(t)\right]\chi_k^{\pm}(t) = 0, \tag{19}$$

subject to $\chi_k^{\pm}(t) = e^{\pm i\omega_{in}t}$ at $t \leq 0$, for incident frequency $\omega_{in} = \sqrt{\mathbf{k}^2 + \epsilon_0 M^2}$, for $m^2(t) = \epsilon(t)M^2$, where $\epsilon(t)$ is parametrised as for the TDLG equation above. This corresponds to a temperature quench from an initial state of thermal equilibrium at temperature $T_0 > T_c$, where $(T_0/T_c - 1) = \epsilon_0$. There is no reason to take ϵ_0 small. The diagonal correlation function $G(r,t)$ of Eq.16 is given as the equaltime propagator

$$G(r,t) = \int đ^3k \, e^{i\mathbf{k}.\mathbf{x}} \chi_k^{+}(t)\chi_k^{-}(t)C(k), \tag{20}$$

where $C(k) = \coth(\omega_{in}(k)/2T_0)/2\omega_{in}(k)$ encodes the initial conditions.

An exact solution can be given [11] in terms of Airy, and related, functions. Dimensional analysis shows that, at displaced time $\Delta t = 0$, $\xi_{eq}(t_C)$ of Eq.5 again sets the scale of the equaltime correlation function. We have yet to work out whether the coefficient is appproximately unity, but Kibble's insight is correct. Further detail is unnecessary since we shall argue that it is the assumption that we can infer the defect density from $\xi_{eq}(t_C)$ that is flawed.

V. DEFECT DENSITIES DO NOT DETERMINE CORRELATION LENGTHS DIRECTLY

We have seen that there is no reason to disbelieve the causal arguments of Kibble for QFT and Zurek for condensed matter as to the time at which the field fluctuations freeze in, and the associated correlation length. The excellent agreement with the ^3He experiments also shows that, for condensed matter, this length does, indeed, characterise vortex separation at the time when the defects form.

However, if we take the Lancaster experiment at face value, this cannot always be the case. If we think about why this might be, the differences between the ^3He and ^4He experiments are twofold. Firstly, the nuclear-driven quench rate for ^3He experiments is several orders of magnitude faster than the mechanically-driven quenches of the ^4He experiments. Secondly, for ^3He the Ginzburg regime is extremely narrow, whereas for ^4He it is very broad. In fact, the ^4He experiments begin and end in the Ginzburg regime, where thermal fluctuations are important. The causality arguments are too simple to accomodate these facts.

If these differences are to be visible in the formalism, it can only be through the way in which we relate vortex density to correlation length. That is, the flaw of the prediction lies in the assumption Eq.11, as becomes apparent once we appreciate that it hides the problem of how to count vortices. One way is to work through line zeroes. For convenience we restrict ourselves to $O(2)$ vortices whose cores are line zeroes of the complex field ϕ. [4] The converse is not true since zeroes occur on all scales. However, a starting-point for counting vortices in superfluids is to count line zeroes of an appropriately coarse-grained field, in which structure on a scale smaller than ξ_0, the classical vortex size, is not present [12]. This is also the unstated basis of the numerous numerical simulations [13] of cosmic string networks built from Gaussian fluctuations (but see [14]). For the moment, we put in a cutoff $l = O(\xi_0)$ by hand, as

$$G_l(r,\Delta t) = \int đ^3k \, e^{i\mathbf{k}.\mathbf{x}} G(k,\Delta t) \, e^{-k^2 l^2}. \tag{21}$$

We stress that the *long-distance* correlation length $\xi_{eq}(t_C)$ depends essentially on the position of the nearest singularity of $G(k,\Delta t)$ in the complex k-plane, *independent* of l.

This is not the case for the line-zero density n_{zero}, depending, in our Gaussian approximation [15,16], on the *short-distance* behaviour of $G_l(r,\Delta t)$ as

$$n_{zero}(\Delta t) = \frac{-1}{2\pi}\frac{G_l''(0,\Delta t)}{G_l(0,\Delta t)}, \tag{22}$$

the ratio of fourth to second moments of $G(k,\Delta t)\, e^{-k^2 l^2}$.

A. TDLG condensed matter

In the linear regime everything is calculable. If we define the line-zero separation ξ_{zero} by

$$n_{zero}(\Delta t) = \frac{1}{2\pi \xi_{zero}(\Delta t)^2} \tag{23}$$

it is apparent that $\xi_{zero}(\Delta t)$ has little, if anything, to do with $\xi_{eq}(\Delta t)$ directly.

[4] More complicated vortices require more complicated analysis, but we expect essentially the same conclusions.

In fact, at $\Delta t = 0$,

$$\xi_{zero}^2 \approx O(l^2), \tag{24}$$

independent of ϵ_0. We have a situation in which the density of line zeroes depends entirely on the scale at which we look. Such fractal behaviour cannot be understood as representing stable vortices, although such line zeroes might be termed *fluctuation* vortices. We would not wish to identify the length scale ξ_{zero} with defect separation at this time, even if the latter could be defined.

This is not surprising. Although the field correlation length $\xi(\Delta t)$ may have frozen in by $\Delta t = 0$, the symmetry breaking has not been effected. In a classical sense at least, vortices can only be identified once the field magnitude has grown to its equilibrium value

$$\langle|\phi|^2\rangle = \alpha_0/\beta, \tag{25}$$

and we should not begin to count them before then. Even though the long-range correlation length $\xi(\Delta t)$ will not change substantially in that time, this will not be the case for $\xi_{zero}(\Delta t)$, as long-range modes increase in amplitude as the field becomes ordered.

Specifically, on continuing to use (15) for times $t > \epsilon_0\tau_Q$ we see that, as the unfreezing occurs, long wavelength modes with $k^2 < t/\tau_Q - \epsilon_0$ grow exponentially. Provided $\epsilon_0^2\tau_Q \gg 1$ they soon begin to dominate the correlation functions. Let

$$G_n(\Delta t) = \int_0^\infty dk\, k^{2n}\, G(k, \Delta t) \tag{26}$$

be the moments of $G(k, \Delta t)$. In units of ξ_0 and τ_0 we find [10] that

$$G_n(\Delta t) \approx \frac{I_n}{2^{n+1/2}}\, e^{(\Delta t/\bar{t})^2} \int_0^\infty dt'\, \frac{e^{-(t'-\Delta t)^2/\bar{t}^2}}{[t' + l^2/2]^{n+1/2}}, \tag{27}$$

where we measure the dimensionless time Δt in units of $\bar{t} = \sqrt{\tau_Q}$ from $t = \epsilon_0\tau_Q$ and $I_n = \int_0 dk k^{2n}\, e^{-k^2}$. For small relative times the integrand gets a large contribution from the ultraviolet *cutoff dependent* lower endpoint, and we recover (24). As long as the endpoints make a significant contribution to the whole then the density of line zeroes derived from (27) will be strongly dependent on scale. Only when their contribution is small and $\partial n_{zero}/\partial l$ is small in comparison to n_{zero}/l at $l = \xi_0$ can we identify the essentially non-fractal n_{zero} with a meaningful vortex density.

Δt_1 and Δt_2, the times at which the exponential modes begin to dominate in the integrands of G_1 and G_2, can be determined by comparing the relative strengths of the contributions from the scale-independent saddlepoint and the endpoint in (27). If $p_1 = \Delta t_1/\bar{t}$ and $p_2 = \Delta t_2/\bar{t}$ are the multiples of $\bar{t}$ at which it happens then, for the former to dominate the latter requires $e^{p_1^2}/p_1^{3/2} > \tau_Q^{1/4}/\sqrt{2\pi}$ and $e^{p_2^2}/p_2^{5/2} > 3\sqrt{2}\tau_Q^{3/4}/\sqrt{\pi}$. We see that it takes longer for the long wavelength modes to dominate G_2 than the order parameter G_1. Because of the exponential growth, p_1 and p_2 are $O(1)$. We stress that these are lower bounds.

If the linear equation (27) were valid for later times $\Delta t > \Delta t_2$ then the integrals are dominated by the saddlepoint at $t' = \Delta t$, to give a separation of line zeroes $\xi_{zero}(\Delta t)$ of the form

$$\xi_{zero}^2(\Delta t) = \frac{G_1(\Delta t)}{G_2(\Delta t)} \approx \frac{4\Delta t}{3\bar{t}}\xi_0^2\left(\frac{\tau_Q}{\tau_0}\right)^{1/2} = \frac{4\Delta t}{3\bar{t}}\xi_{eq}(t_C)^2 \tag{28}$$

approximately *independent* of the cutoff l for $l = O(\xi_0)$. Only then, because of the transfer of power to long wavelengths, do line zeroes become widely separated, and $\xi_{zero}(\Delta t)$ does begin to measure vortices. Thus, if the order parameter is large enough that it takes a long time (in units of $\bar{t}$) for the field to populate its ground states (25) we would recover the result $\xi_{zero} = O(\xi_{eq}(t_C))$ as an order of magnitude result from (28) even though, *a priori*, they are unrelated. This explains how experiments can be in agreement with the simple causal predictions.

Whether we have time enough depends on the self-coupling β, which determines when the linear approximation fails. At the absolute latest, the correlation function must stop its exponential growth at $\Delta t = \Delta t_{sp}$, when $\langle|\phi|^2\rangle$, proportional to G_1, satisfies (25). Let us suppose that the effect of the backreaction that stops the growth initially freezes in any defects. This then is our prospective starting point for identifying and counting vortices.

It happens that (25) is satisfied when

$$G_1(\Delta t_{sp}) = \pi^2\alpha_0^2\xi_0^3/\beta k_B T_c = \pi^2/\sqrt{1 - T_G^-/T_c}, \tag{29}$$

where T_G^- is the Ginzburg temperature introduced earlier.

In order that G_1 is dominated by the exponentially growing modes, so as to recover the modification of prediction (11) in the form (28) for identifiable vortices, the condition $\Delta t_{sp} > \Delta t_1, \Delta t_2$ becomes, from (29)

$$(\tau_Q/\tau_0)(1 - T_G^-/T_c) < C\pi^4, \tag{30}$$

on restoring τ_0, where $C \approx 1$.

How strongly the inequality should be satisfied in (30) is not obvious, assuming as it does that the backreaction is effectively instantaneous, and that mean field critical indices are valid. However, there is no way that the inequality can be remotely satisfied for ^4He, when subjected to a slow mechanical quench, as in the Lancaster experiment, for which $\tau_Q/\tau_0 = O(10^{10})$, since the Ginzburg regime is so large that $(1 - T_G/T_c) = O(1)$. That is, field growth must stop long before vortices are well defined. Further, since the experiment leaves the superfluid in the Ginzburg regime, thermal fluctuations will inhibit the creation of stable defects, although it may be that incoherent fluctuations, even if not vortices, will

give a signal. Adopting renormalisation improved critical indices cannot repair such a deficit. Whatever, there is no reason to expect a vortex density (22).

The situation for ^3He-B is potentially very different, For rapid quenches a TDLG approach is valid [17]. Suppose that (30), which looks like an inequality between the quench rate and the equilibriation rate that permits unstable modes time to dominate over short-range fluctuations, remains true, albeit with new coefficients. Firstly, for ^3He-B, the Ginzburg regime is very small, with $1-T_G/T_c = O(10^{-8})$. The quench ends outside it. Secondly, in generating the phase transition by nuclear reactions, rather than by mechanical expansion, the quench rate is increased dramatically, with $\tau_Q = 10^2 - 10^4$. The inequality (30) is satisfied by a huge margin and we can understand the success of the Helsinki and Grenoble experiments.

B. QFT: mode growth v fluctuations

It is because the formation of defects is an early-time occurrence that it is, in large part, amenable to analytic solution. Again we revert to the mode decomposition of Eq.19. The field becomes ordered, as before, because of the exponential growth of long-wavelength modes, which stop growing once the field has sampled the groundstates. What matters is the relative weight of these modes (the 'Bragg' peak) to the fluctuating short wavelength modes in the decomposition Eq.20 at this time, since the contribution of these latter is very sensitive to the cutoff l. Only if their contribution to Eq.11 is small when field growth stops can a network of vortices be well-defined at early times, let alone have the predicted density. Since the peak is non-perturbatively large this requires small coupling, which we assume.

However, to determine the counterpart to Eq.30 for QFT is not so simple, and it helps to begin with an idealised problem, in which the quench is *instantaneous* ($t_Q = 0$), and $m^2(t)$ flips from $+\epsilon_0 M^2$ to $-M^2$ at $t = 0$. Eq.11 is clearly inappropriate, a useful warning that the simple picture above has its limitations. However, once we have understood the instantaneous case the slow quench is not much more difficult.

The mode equations Eq.19 are instantly solvable, and $G(r,t)$ of Eq.20 is simple calculable.

Rather than introduce a cutoff at scale l, $l^{-1} = \Lambda = O(M)$, as in Eq.21, we adopt the more brutal approach of cutting off modes at $|\mathbf{k}| < \Lambda$ in Eq.20. Imposing the KMS boundary conditions at $t \leq 0$ for an initial Boltzmann distribution at temperature T_0 compatible with thermal mass $\sqrt{\epsilon_0}M$ determines the resulting coarse-grained $G_\Lambda(r;t)$ uniquely. For $t < 0$ all modes are oscillatory, but for $t > 0$ long wavelength modes with $k^2 > \Lambda^2$ are unstable, and grow exponentially. For $\Lambda = O(M)$, but $\Lambda > M$, say, $G_\Lambda(r;t)$ can be decomposed in an obvious way as $G_\Lambda(r;t) = G_\Lambda^{in}(r)$ for $t \leq 0$, and

$$G_\Lambda(r;t) = G_\Lambda^{in}(r) + G_{|\mathbf{k}|<M}(r;t) + G_{\Lambda>|\mathbf{k}|>M}(r;t) \quad (31)$$

for $t > 0$, where [18] the second and third terms are the long and short wavelength fluctuations that grow from $t = 0$ onwards.

The equilibrium background term $G_\Lambda^{in}(r)$ has the form

$$G_\Lambda^{in}(r) = \int_{|\mathbf{k}|<\Lambda} d\!\!{}^-{}^3k\, e^{i\mathbf{k}.\mathbf{x}} C(k), \quad (32)$$

where $C(k)$ encodes the initial conditions as before. The correlation length of the field is $\xi_0 = M^{-1}$ and, for cutoff $\Lambda = O(M)$, the cold vortex thickness, we have line zero density $n_{zero} = O(\Lambda^2)$, as in the condensed matter case. There is a sea of line zeroes, largely tiny loops, separated only by the Compton wavelength. The oscillating field fluctuations make them unsuitable as serious candidates for string since the density depends critically on the scale Λ^{-1} at which we view the field.

For early positive times n_{zero} is equally sensitive to Λ but, once $Mt \gg 1$, the relevant term is $G_{|\mathbf{k}|<M}(r;t)$, whose integral at time t is dominated by the nonperturbatively large peak [18] in the power of the fluctuations at k around k_0, where $tk_0^2 \simeq M$. Once $k_0 \ll M$ we have the required insensitivity of the line density to the scale at which we coarsegrain. We find, approximately, that

$$G_{|\mathbf{k}|<M}(r;t) \propto \frac{MT_0}{(tM)^{3/2}} e^{2Mt} \exp\{-r^2 M/4t\}. \quad (33)$$

The prefactor comes from approximating $C(k_0)$ by $T_0/\epsilon_0 M^2$. For $t > 0$ the unstable modes grow until $G_{|\mathbf{k}|<M}(0;t_{sp}) = O(\phi_0^2) = O(M^2/\lambda)$, determining the spinodal time t_{sp} as $Mt_{sp} = O(\ln(1/\lambda))$. The long-distance behaviour of $G_\Lambda^{in}(r) = O(e^{-Mr})$, with its correlation length $\xi = M^{-1}$ is a shortlived relic of the initial thermal conditions. After a time $t_r = O(M^{-1}) \leq t_{sp}$, it is rapidly supplanted by the behaviour of the expanding long wavelength modes $G_{|\mathbf{k}|<M}(r;t)/G_{|\mathbf{k}|<M}(0;t) \approx \exp\{-r^2/\xi^2(t)\}$ where $\xi^2(t) = 4t/M \approx 4/k_0^2$. With $G_{|\mathbf{k}|<M}(0;t_{sp}) = O(\lambda^{-1})$ non-perturbatively large and $G_\Lambda^{in}(0)$ and $G_{\Lambda>|\mathbf{k}|>M}(0;t_{sp})$ of order $\lambda^{-1/2}$,

$$n_{zero}(t_{sp}) \approx \frac{1}{\pi\xi_0^2 \ln(1/\lambda)}[1 + O(\lambda^{1/2}\ln(1/\lambda))]. \quad (34)$$

The relative error term in the brackets is, in large part, a measure of the fluctuation vortices that we mentioned earlier and is a measure of the stability of this density to changes in the coarse-graining scale $\Lambda = O(M)$. For weak coupling $O(\lambda^{1/2}\ln(1/\lambda)) \ll 1$.

There are other, less direct, ways to understand why the strings only become well-defined once $G_M(0;t)$ is nonperturbatively large, even though the density is independent of its *magnitude*. Instead of a field basis, we can work in a particle basis and measure the particle production as the transition proceeds. Whether we expand with respect to the original Fock vacuum or with respect to the adiabatic vacuum state, the presence of a

non-perturbatively large peak in $k^2G(k;t)$ signals a non-perturbatively large occupation number $N_{k_0} \propto 1/\lambda$ of particles at the same wavenumber k_0 [18]. With n_{zero} of (34) of order k_0^2 this shows that the long wavelength modes can now begin to be treated classically.

From a slightly different viewpoint, the Wigner functional only peaks about the classical phase-space trajectory once the power is non-perturbatively large [19,20] from time t_{sp} onwards. More crudely, the diagonal density matrix elements (field probabilities) are only then significantly non-zero for non-perturbatively large field configurations $\phi \propto \lambda^{-1/2}$ like vortices.

So far we have done no more than estimate the density of coarse-grained line zeroes for a free roll at the time t_{sp} at which nothing has frozen in. For this *global* $O(2)$ theory the damping of domain growth occurs by the self-interaction effectively forcing the negative $m^2(t)$ to vanish so as to produce Goldstone particles. This initial slowdown leads to no qualitative change in the vortex density.

All the results above were for the instantaneous quench. We now return to the original problem of slower quenches, with $\epsilon(t)$ as in Eq.14, in which the symmetry-breaking begins at relative time $\Delta t = 0.$. For a *free* roll, the exponentially growing modes that appear when $\Delta t > t_k^- = t_Q k^2/M^2$ lead to the approximate solution [21]

$$G(r;\Delta t) \propto \frac{T}{M|m(\Delta t)|}\left(\frac{M}{\sqrt{\Delta t t_Q}}\right)^{3/2} e^{\frac{4M\Delta t^{3/2}}{3\sqrt{t_Q}}}\, e^{-r^2/\xi^2(\Delta t)} \tag{35}$$

where $\xi^2(\Delta t) = 2\sqrt{\Delta t t_Q}/M$. The provisional relative freeze-in time Δt_{sp} is then, for $Mt_Q < (1/\lambda)$,

$$M\Delta t_{sp} \simeq (Mt_Q)^{1/3}(\ln(1/\lambda))^{2/3} \simeq Mt_C(\ln(1/\lambda))^{2/3}. \tag{36}$$

This is greater than Mt_C, but not by a large multiple unless we have a superweak theory. More detail is given in [21].

At this qualitative level the correlation length at the spinodal time is

$$M^2\xi^2(t_{sp}) \simeq (Mt_Q)^{2/3}(\ln(1/\lambda))^{1/3}. \tag{37}$$

The effect of the other modes is larger than for the instantaneous quench, giving, at $t = t_{sp}$

$$n_{zero} = \frac{M^2}{\pi(M\tau_Q)^{2/3}}(\ln(1/\lambda))^{-1/3}[1+E]. \tag{38}$$

The error term $E = O(\lambda^{1/2}(Mt_Q)^{4/3}(\ln(1/\lambda))^{-1/3})$ is due to fluctuation vortices. In mimicry of Eq.11 it is helpful to rewrite Eq.38 as

$$n_{zero} = \left[\frac{1}{\pi\xi_0^2}\left(\frac{\tau_0}{\tau_Q}\right)^{2/3}\right](\ln(1/\lambda))^{-1/3}[1+E]. \tag{39}$$

in terms of the scales $\tau_0 = \xi_0 = M^{-1}$. The first term in Eq.39 is the Kibble estimate of Eq.11, the second is the small multiplying factor, rather like that in Eq.28, that yet again shows that estimate can be correct, but for completely different reasons. The third term shows when it can be correct, since E is also a measure of the sensitivity of n_{zero} to the scale at which it is measured. We note that if we take $M\tau_Q = \ln(1/\lambda)$ we recover the instantaneous results qualitatively. Only for larger τ_Q will the quench give different results. The condition $E^2 \ll 1$, necessary for a vortex network to be defined, is then guaranteed if

$$(\tau_Q/\tau_0)^2(1 - T_G^-/T_c) < 1, \tag{40}$$

on using the relation $(1 - T_G^-/T_c) = O(\lambda)$. This is the QFT counterpart to Eq.30.

The density of Eq.39 is *smaller* than that of Eq.38 by the factor $(\ln(1/\lambda))^{1/3}$ which is, in principle, a large number even if in practice it is only a small multiple. The easiest way to enforce $E \ll 1$ and $M\tau_Q > \ln(1/\lambda)$ is to take $M\tau_Q = \ln(1/\lambda)^\alpha$, for $\alpha > 1$. The effect in Eq.39 is merely to renormalise the critical index. Of course, the Kibble prediction Eq.11 was only an estimate. Although it is good qualitatively it is misleading when considering definition of the network since a simple calculations shows that, at time t_C, the string density is still XStotally sensitive to the definition of coarse-graining.

Finally, suppose that this approach is relevant to the local strings of a strong Type-II $U(1)$ theory for the early universe, in which the time-temperature relationship $tT^2 = \Gamma M_{pl}$ is valid, where we take $\Gamma = O(10^{-1})$ in the GUT era. If G is Newton's constant and μ the classical string tension then, following [3], $Mt_Q \sim 10^{-1}\lambda^{1/2}(G\mu)^{-1/2}$. The dimensionless quantity $G\mu \sim 10^{-6} - 10^{-7}$ is the small parameter of cosmic string theory. A value $\lambda \sim 10^{-2}$ gives $Mt_Q \sim (Mt_{sp})^a, a \sim 2$, once factors of π, etc.are taken into account, rather than $Mt_Q \sim 1/\lambda$, and the density of Eq.39 may be relevant.

We have not attempted to show what happens directly after the formation of defects for those cases when they are well-defined by the time the initial population of the ground states has occurred. We have only attempted to test the Kibble/Zurek scenarios for defect formation. For strongly-damped systems like condensed matter systems the linear approximation seems to be valid for longer times than might have been guessed. That would explain why the final population is simply related to the initial population. For QFT it is much less obvious that this is the case. That is, even when Kibble is correct, the later evolution may make the estimate irrelevant. To take one extreme example. If we suppose that the field evolution can be approximated by *self-consistent* linearisation then, very rapidly, the scattering between field modes, absent in the linear approximation here, will be enough to redistribute power back to short wavelengths. The density of line zeroes then begins to *increase* rapidly. If they do represent strings their density is likely to be independent of τ_Q [22].

VI. CONCLUSIONS

The Kibble/Zurek scenario for the onset of a continuous transition, either in QFT or condensed matter, suggests that causality determines the correlation length of the fields at the time that they freeze in. Our simple calculations show that this is qualitatively correct.

A further step in their arguments was to identify this calculable correlation length with the length of defect separation (cosmic strings or vortices) at the time at which such defects were produced. Whereas this is in agreement with vortex experiments on ^3He it is not the case for ^4He. In order to understand this we have identified defect density with field-zero density, once this is effectively independent of the scale at which the zeroes are counted.

The separation of zeroes is determined from the short-distance field correlations and the correlation length (by definition) from the long-distance field correlation function. Nonetheless, the field correlation length at the time it freezes in and the zero separation length when the initial relaxation of the fields to their ground states is complete are qualitatively the same, *provided* thermal fluctuations are not too strong for a well-defined vortex network to be established. Qualitatively, this requires (in the terminology of the text)

$$(\tau_Q/\tau_0)^{\gamma}(1 - T_G^-/T_c) < 1, \tag{41}$$

where (in mean field) $\gamma = 1$ for condensed matter, and $\gamma = 2$ for QFT.

This linking of quench rate to the Ginzburg regime explains why ^3He and ^4He experiments are so different, and why we expect the simple predictions for vortex densities not to be verified in the latter. A constraint like Eq.41 is not strong enough to preclude initial defect formation in the early universe.

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[1] T.W.B. Kibble, *J. Phys.* **A9**, 1387 (1976).

[2] T.W.B. Kibble, in *Common Trends in Particle and Condensed Matter Physics*, *Physics Reports* **67**, 183 (1980).

[3] W.H. Zurek, *Nature* **317**, 505 (1985), *Acta Physica Polonica* **B24**, 1301 (1993). See also W.H. Zurek, *Physics Reports* **276**, Number 4, November 1996

[4] W.H. Zurek, *Nature* **382**, 297 (1996),
P. Laguna and W.H. Zurek, *Phys. Rev. Lett.* **78**, 2519 (1997). A. Yates and W.H. Zurek, hep-ph/9801223.

[5] M.M. Salomaa and G.E. Volovik, *Rev. Mod. Phys.* **59**, 533 (1987)

[6] V.M.H. Ruutu *et al.*, *Nature* **382**, 334 (1996).

[7] C. Bauerle *et al.*, *Nature* **382**, 332 (1996).

[8] P.C. Hendry *et al*, *Nature* **368**, 315 (1994).

[9] P.V.E McClintoch *et al.*, to be published in *Phys. Rev. Letters.*

[10] G. Karra and R. J. Rivers, to be published in *Phys. Rev. Letters.*

[11] M. Bowick and A. Momen, preprint hep-ph/9803284, to be published.

[12] V.N. Popov *Functional integrals and collective excitations*, Cambridge University Press (1997), Chapter 8.

[13] T. Vachasparti and A. Vilenkin, *Phys. Rev.* **D30**, 2036 (1984), A.Vilenkin and R.J. Scherrer, *Phys. Rev.* **D5** 647 (1997), see also preprint hep-ph/9709498.

[14] M. Gleiser and H-R. Muller, preprint hep-lat/9704005, to be published in *Physics Letters B.*

[15] B.I. Halperin, published in *Physics of Defects*, proceedings of Les Houches, Session XXXV 1980 NATO ASI, editors Balian, Kléman and Poirier (North-Holland Press, 1981) p.816.

[16] F. Liu and G.F. Mazenko, *Phys. Rev.* **B46**, 5963 (1992).

[17] Yu. M. Bunkov and O.D. Timofeevskaya, preprint cond-mat/9706004, *Phys. Rev. Letters*, to be published.

[18] D. Boyanovsky, Da-Shin Lee and A. Singh, *Phys. Rev.* **D48**, 800 (1993),
D. Boyanovsky, H.J. de Vega and R. Holman, *Phys. Rev.* **D49**, 2769 (1994).

[19] A. Guth and S-Y. Pi, *Phys. Rev.* **D32**, 1899 (1985). O. Eboli, R. Jackiw and S-Y. Pi, *Phys. Rev.* **D37**, 3557 (1988).

[20] S. Mrowczynski and B. Muller, *Phys. Rev.* **D50**, 7542 (1994).

[21] G. Karra and R.J. Rivers, *Physics Letters* **B414**, 28 (1997)

[22] H.J. de Vega et al., private communication.